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Deviation from Tetra-Maximal Neutrino Mixing Above the GUT scale

Bipin Singh Koranga · P. Gaira

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Abstract We consider non-renormalizable interaction term as perturbation of the conventional neutrino mass matrix. We assume that the neutrino masses and mixing arise through physics at a scale intermediate between Planck scale and the electroweak breaking scale. We also assume that, just above the electroweak breaking scale, neutrino masses are nearly degenerate and their mixing is tetra-maximal. Quantum gravity (Planck scale effects) lead to an effective $SU(2)_L \times U(1)$ invariant dimension-5 Lagrangian involving neutrino and Higgs fields. On electroweak symmetry breaking, this operator gives rise to correction to the above masses and mixing. These additional term can be consider as a perturbation to the Tetra-maximal mass matrix. The nature of gravitational interaction demands that the element of this perturbation matrix should be independent of flavor indices. We compute the deviation of three neutrino mixing angles due to Planck scale effects. We find that there is no change in θ_{13} and θ_{23} but change in solar mixing angle θ_{12} is suppress by 3.0°.

1 Introduction

Recent current oscillation data [1–6] indicates a peculiar pattern of neutrino mass and mixing quiet at variance with the structure of the CKM quark mixing matrix [7]. Recent observed neutrino oscillation data are in favor of the so called "tribimaximal mixing" [8], which predict $\theta_{13} = 0$, $\tan\theta_{12} = 1/3$ and $\sin^2 2\theta_{23} = 1.0$, since present data yield the $\tan^2\theta_{12} = 0.47^{+0.006}_{-0.05}$ [9] and $\sin^2 2\theta_{23} = 1.00_{-0.13}$ [10]. The mixing of lepton flavors is described by a 3×3 unitary matrix U , whose nine elements are parametrized in term of three rotation angle and three CP violating phases. The standard parametrization of U expressed by the Particle Data Group [11]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{23}s_{13} \end{pmatrix} P, \tag{1}$$

B.S. Koranga (✉) · P. Gaira
Department of Physics, Kirori Mal College, University of Delhi, Delhi 110007, India
e-mail: bipiniitb@rediffmail.com

where $P = \text{diag}(e^{ia1}, e^{ia2}, 1)$ is a diagonal matrix, which contain two Majorana phase of CP violation. The forth coming neutrino oscillation experiment will measure θ_{13} and δ. On the other hand, the neutrinoless double beta decay experiment will help to constrain $a1$ and $a2$. The new mixing pattern [12], which will referred to as the "Tetra-maximal" neutrino mixing gives

$$\begin{aligned}\theta_{12} &= \arctan\left[2\left(1-\sqrt{\frac{1}{2}}\right)\right] \approx 34.4^\circ,\\ \theta_{13} &= \arcsin\left[2\left(1-\sqrt{\frac{1}{2}}\right)\right] \approx 8.4^\circ,\\ \theta_{23} &= 45^\circ\end{aligned} \tag{2}$$

and $\delta = 90^\circ$ together with $a1 = a2 = -90^\circ$. Tetra-maximal scenero is easily testable in neutrino oscillation experiment in the near future. In Sect. 2, we outline the neutrino mixing due to Quantum gravity. In Sect. 3, we compute the neutrino mixing from Planck scale effects. In Sect. 4 is devoted to the conclusions.

2 Neutrino Oscillation Parameter due to Planck Scale Effects

The neutrino mass matrix is assumed to be generated by the see saw mechanism [13–15]. We assume that the dominant part of neutrino mass matrix arise due to GUT scale operators and the lead to tetra-maximal mixing. The effective gravitational interaction of neutrino with Higgs field can be expressed as $SU(2)_L \times U(1)$ invariant dimension-5 operator [22],

$$L_{\text{grav}} = \frac{\lambda_{\alpha\beta}}{M_{pl}}(\psi_{A\alpha}\epsilon_{AC}\psi_C)C^{-1}_{ab}(\psi_{B\beta}\epsilon_{BD}\psi_D) + h.c. \tag{3}$$

Here and every where we use Greek indices α, β for the flavor states and Latin indices i, j, k for the mass states. In the above equation $\psi_\alpha = (\nu_\alpha, l_\alpha)$is the lepton doublet, $\phi = (\phi^+, \phi^o)$is the Higgs doublet and $M_{pl} = 1.2 \times 10^{19}$ GeV is the Planck mass λ is a 3×3 matrix in a flavor space with each elements $O(1)$. The Lorentz indices $a, b = 1, 2, 3, 4$ are contracted with the charge conjugation matrix C and the $SU(2)_L$ isospin indices $A, B, C, D = 1, 2$ are contracted with $\epsilon = i\sigma_2$, σ_m $(m = 1, 2, 3)$ are the Pauli matrices. After spontaneous electroweak symmetry breaking the Lagrangian in (3) generated additional term of neutrino mass matrix

$$L_{\text{mass}} = \frac{v^2}{M_{pl}}\lambda_{\alpha\beta}\nu_\alpha C^{-1}\nu_\beta, \tag{4}$$

where $v = 174$ GeV is the VEV of electroweak symmetric breaking. We assume that the gravitational interaction is "flavor blind" that is $\lambda_{\alpha\beta}$ is independent of α, β indices. Thus the Planck scale contribution to the neutrino mass matrix is

$$\mu\lambda = \mu\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}, \tag{5}$$

where the scale μ is

$$\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6}\ \text{eV}. \tag{6}$$

We take (5) as perturbation to the main part of the neutrino mass matrix, that is generated by GUT dynamics. To calculate the effects of perturbation on neutrino observables. The calculation developed in an earlier paper [15]. A natural assumption is that unperturbed (0th order mass matrix) M is given by

$$\mathbf{M} = U^* \text{diag}(M_i) U^\dagger, \tag{7}$$

where, $U_{\alpha i}$ is the usual mixing matrix and M_i, the neutrino masses is generated by Grand unified theory. Most of the parameter related to neutrino oscillation are known, the major expectation is given by the mixing elements U_{e3}. We adopt the usual parametrization.

$$\frac{|U_{e2}|}{|U_{e1}|} = \tan\theta_{12}, \tag{8}$$

$$\frac{|U_{\mu 3}|}{|U_{\tau 3}|} = \tan\theta_{23}, \tag{9}$$

$$|U_{e3}| = \sin\theta_{13}. \tag{10}$$

In term of the above mixing angles, the mixing matrix is

$$U = \text{diag}(e^{if1}, e^{if2}, e^{if3}) R(\theta_{23}) \Delta R(\theta_{13}) \Delta^* R(\theta_{12}) \text{diag}(e^{ia1}, e^{ia2}, 1). \tag{11}$$

The matrix $\Delta = \text{diag}(e^{\frac{1\delta}{2}}, 1, e^{\frac{-i\delta}{2}})$ contains the Dirac phase. This leads to CP violation in neutrino oscillation $a1$ and $a2$ are the so called Majoring phase, which effects the neutrino less double beta decay. $f1$, $f2$ and $f3$ are usually absorbed as a part of the definition of the charge lepton field. Planck scale effects will add other contribution to the mass matrix that gives the new mixing matrix can be written as [15]

$$\begin{aligned} U^{'} &= U(1 + i\delta\theta), \\ &\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\ &+ i \begin{pmatrix} U_{e2}\delta\theta_{12}^* + U_{e3}\delta\theta_{23,}^* & U_{e1}\delta\theta_{12} + U_{e3}\delta\theta_{23}^*, & U_{e1}\delta\theta_{13} + U_{e3}\delta\theta_{23}^* \\ U_{\mu 2}\delta\theta_{12}^* + U_{\mu 3}\delta\theta_{23,}^* & U_{\mu 1}\delta\theta_{12} + U_{\mu 3}\delta\theta_{23}^*, & U_{\mu 1}\delta\theta_{13} + U_{\mu 3}\delta\theta_{23}^* \\ U_{\tau 2}\delta\theta_{12}^* + U_{\tau 3}\delta\theta_{23}^*, & U_{\tau 1}\delta\theta_{12} + U_{\tau 3}\delta\theta_{23}^*, & U_{\tau 1}\delta\theta_{13} + U_{\tau 3}\delta\theta_{23}^* \end{pmatrix}, \end{aligned} \tag{12}$$

where $\delta\theta$ is a hermition matrix that is first order in μ [15–17]. The first order mass square difference $\Delta M_{ij}^2 = M_i^2 - M_j^2$,get modified [15–17] as

$$\Delta M_{ij}^{'2} = \Delta M_{ij}^2 + 2(M_i \,\text{Re}(m_{ii}) - M_j \,\text{Re}(m_{jj}), \tag{13}$$

where

$$m = \mu U^t \lambda U,$$

$$\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6}\ \text{eV}.$$

The change in the elements of the mixing matrix, which we parametrized by $\delta\theta$ [15–17], is given by

$$\delta\theta_{ij} = \frac{i\,\mathrm{Re}(m_{jj})(M_i + M_j) - \mathrm{Im}(m_{jj})(M_i - M_j)}{\Delta M_{ij}^{'2}}. \tag{14}$$

The above equation determine only the off diagonal elements of matrix $\delta\theta_{ij}$. The diagonal element of $\delta\theta_{ij}$ can be set to zero by phase invariance. Using (12), we can calculate neutrino mixing angle due to Planck scale effects,

$$\frac{|U_{e2}^{'}|}{|U_{e1}^{'}|} = \tan\theta_{12}^{'}, \tag{15}$$

$$\frac{|U_{\mu 3}^{'}|}{|U_{\tau 3}^{'}|} = \tan\theta_{23}^{'}, \tag{16}$$

$$|U_{e3}^{'}| = \sin\theta_{13}^{'}. \tag{17}$$

For degenerate neutrinos, $M_3 - M_1 \cong M_3 - M_2 \gg M_2 - M_1$, because $\Delta_{31} \cong \Delta_{32} \gg \Delta_{21}$. Thus, from the above set of equations, we see that $U_{e1}^{'}$ and $U_{e2}^{'}$ are much larger than $U_{e3}^{'}$, $U_{\mu 3}^{'}$ and $U_{\tau 3}^{'}$. Hence we can expect much larger change in θ_{12} compared to θ_{13} and θ_{23}. As one can see from the above expression of mixing angle due to Planck scale effects, depends on new contribution of mixing matrix $U^{'} = U(1 + i\delta\theta)$. New contribution of mixing matrix also changes the electromagnetic properties and CP symmetry of neutrinos [18–20].

3 Numerical Results

We assume that, just above the electroweak breaking scale , the neutrino masses are nearly degenerate and the mixing are tetra-maximal, with the values of the mixing angle as $\theta_{12} = 34.4°$, $\theta_{23} = 45°$ and $\theta_{13} = 8.4°$. Taking the common degenerate neutrino masses to be 2 eV, which is the upper limit coming from tritium beta decay [21]. We compute the modified mixing angle using (15) to (17). We have taken $\Delta_{31} = 0.002\ \mathrm{eV}^2$ [23] and $\Delta_{31} = 0.0008\ \mathrm{eV}^2$ [24]. For simplicity we have taken set the charge lepton phase $f_1 = f_2 = f_3 = 0$ mixing matrix. In Table 1, we list the modified neutrino mixing angles. As shown in the table the deviation in θ_{13} and θ_{23} are negligible small but change in solar mixing angle $\delta\theta_{12}$ is 2.5°. Tetra-maximal mixing scenario which predicts $\theta_{12} = \arctan[2(1 - \sqrt{\frac{1}{2}})] \approx 34.4°$, $\theta_{13} = \arcsin[2(1 - \sqrt{\frac{1}{2}})] \approx 8.4°$, $\theta_{23} = 45°$. Planck scale effects give rise to correction to neutrino mass matrix on electroweak symmetry breaking. It is imperative to check that their correction do not spoil the good agreement between the experimental fit and the prediction of Tetra-maximal scenario. We compute the modified mixing angles for this scenario, that is for the input value $\theta_{12} = 34.4°$, $\theta_{13} = 8.4°$ and $\theta_{23} = 45°$. We see that the maximum possible deviation in θ_{12} is about $-3.0°$. Thus, we see that the correction to neutrino mass matrix arising from Planck scale effects do not spoil the agreement between the experiment and the prediction of Tetra-maximal mixing.

4 Conclusions

We assume that the main part of neutrino masses and mixing from GUT scale operator. We considered these to be 0th order quantities. The gravitational interaction of lepton field

Table 1 Deviation from tetra-maximal neutrino mixing. Input value are $\Delta_{31} = 0.002$ eV^2, $\Delta_{21} = 0.00008$ eV^2, $\theta_{12} = 34.4°$, $\theta_{23} = 45°$, $\theta_{13} = 8.4°$, $\delta = 90°$, $a1 = a2 = -90°$

θ'_{12}	$\delta\theta_{12} = \theta'_{12} - \theta_{12}$	θ_{13}	$\delta\theta_{13} = \theta'_{13} - \theta_{13}$	θ_{23}	$\delta\theta_{23} = \theta'_{23} - \theta_{23}$
31.4°	−3.0°	8.4°	0°	45°	0°

with SM Higgs field give rise to a $SU(2)_L \times U(1)$ invariant dimension-5 effective Lagrangian give originally by Weinberg [22]. On electroweak symmetry breaking this operators leads to additional mass terms. We considered these to be perturbation of GUT scale mass terms. We compute the first order correction to neutrino mass eigen value and mixing angles. It was shown that there is no change in θ_{13}, θ_{23} but the change in θ_{12} can be substantial about $-3.0°$.The change in all the mixing angle are proportional to the neutrino mass eigenvalues. To maximizes the change, we assumed degenerate neutrino mass 2.0 eV. For degenerate neutrino masses, the change in θ_{13}, θ_{23} are inversely proportional to Δ_{21}. Since $\Delta_{31} \cong \Delta_{32} \gg \Delta_{21}$ the change in $\theta_{12}$is much larger than the change in other mixing angle. Cosmological constraints, from WMAP measurements [25], impose value of $\theta_{12}$is correspondingly smaller. However, this decrease can be compensated if there is some flavor independent new physics at a scale below Planck scale.

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